SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2007

ADDITIONAL MATHEMATICS HIGHER GRADE

Time: 3 hours 400 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 14 pages and an insert of 4 pages (i – iv) containing formula sheets and Table 1 for Section D. Please check that your question paper is complete.

2. This question paper consists of FOUR sections.

3. **Section A** is compulsory and must be answered by ALL candidates.

4. **Candidates** must also answer any other TWO sections from Sections B, C and D. (Only TWO sections will be marked.)

5. **Each section must be done in a separate answer book and the relevant section must be indicated on the cover**. Place all other answer books in the book for Section A before handing in.

6. Non-programmable calculators may be used, unless otherwise indicated.

7. All necessary calculations must be shown clearly and writing should be legible.

SECTION A CALCULUS

QUESTION 1

(a) $f(x) = \dfrac{3x^2 - 4x + 1}{1 - x^2}$

Evaluate:

(i) $\lim_{x \to 0} f(x)$ (2)

(ii) $\lim_{x \to 1} f(x)$ (4)

(iii) $\lim_{x \to \infty} f(x)$ (4)

(b) Evaluate the limit, giving your answer in simplest surd form:

$$\lim_{h \to 0} \frac{\sqrt{(x+h)^2 + 1} - \sqrt{x^2 + 1}}{h}$$ (7)

17 marks

QUESTION 2

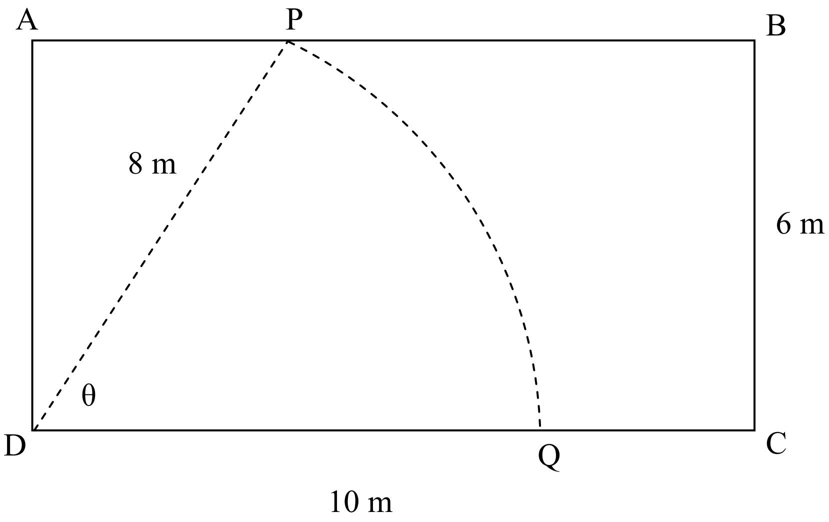
The graph of f is defined as follows:

$$f(x) = \begin{cases} 3x + 15 & \text{if} \quad x < -2 \\ 3ab + 3 & \text{if} \quad x = -2 \\ ax^2 + b & \text{if} \quad x > -2 \end{cases}$$

If it is given that $f(x)$ is continuous at $x = -2$, solve for a and b.

16 marks

QUESTION 3



A horse is tied to a point D in a rectangular field ABCD by means of a rope which is 8 metres long. BC = 6 metres, CD = 10 metres and $P\hat{D}Q = \theta$. The horse can eat grass at any point in shape APQD, where P lies on AB and Q lies on CD.

(a) Find θ, giving your answer in radians. (4)

(b) Calculate the area of the grass QCBP which **cannot** be eaten by the horse. (10)

14 marks

QUESTION 4

$f(x) = x^2$
$g(x) = 3x - 2$

Find the values of a and b if

(a) $(fog)(a) = 100$ (4)

(b) $(fog)(b) = (gof)(b)$ (6)

10 marks

QUESTION 5

Consider the cubic function $f(x) = ax^3 + bx^2 + cx + d$.

(a) Write down expressions for $f'(x)$ and $f''(x)$. (4)

(b) By referring to the graphs of f' and f'', explain why a cubic graph must always have one point of inflection but may not always have a local maximum or minimum value stationary point. (4)

(c) Write down any cubic function which does not have a stationary point and sketch this graph, showing intercepts on the axes. (6)

14 marks

QUESTION 6

(a) Find $f'(4)$ if $f(x) = \sqrt{\sqrt{6x+12} - \sqrt{x}}$ (10)

(b) If $f(x) = \dfrac{10 - \cos x}{x}$

show that $x.f'(x) + f(x) = \sin x$ (10)

20 marks

QUESTION 7

(a) In 1671, the Scottish mathematician, John Gregory, showed that $\arctan x$ could be expressed as the infinite series:

$$\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \frac{x^9}{9} - \text{...........};\ -1 \le x \le 1$$

Use this result to show that the derivative of $\arctan x$ is $\dfrac{1}{1+x^2}$. (10)

(b) Given the function $f(x) = x^3 - 3x^2 - 9x - 8$

(i) Draw a sketch graph of $f(x)$, showing only the shape of the curve, the y-intercept and the x-coordinates of the stationary points. (6)

(ii) Find a suitable initial approximation, and then use the Newton-Raphson formula to find the largest root of $f(x)$ correct to 4 decimal digits. (8)

24 marks

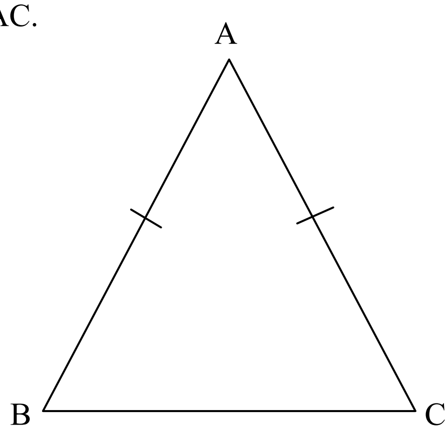
QUESTION 8

If $f(x) = \frac{3}{x}$, determine a formula for the n^{th} derivative of $f(x)$.

10 marks

QUESTION 9

In ΔABC, $AB = AC$.



(a) Show that $\cos A = -\cos 2B$ (6)

(b) Hence, or otherwise, determine the maximum value of $\cos A + \cos B$. (14)

20 marks

QUESTION 10

Find the possible values of k if

$$\int_{-1}^{2k} \left(4 - 3x^2\right) dx = 3$$

10 marks

QUESTION 11

Determine the integrals

(a) $\int (x+2)\sqrt{x^2+4x+5}\ dx$ (9)

(b) $\int \dfrac{x}{\sec 2x} dx$ (11)

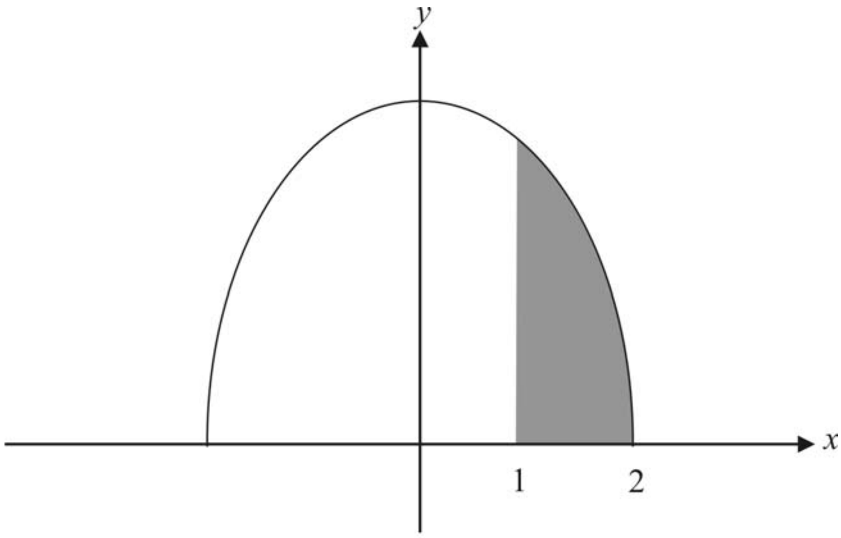
20 marks

QUESTION 12

(a) Use the substitution $x = 2\sin\theta$ to show that:

$\int \sqrt{4-x^2}\ dx = \sin 2\theta + 2\theta + C$ (14)

(b) Part of the ellipse with equation $y = \sqrt{16-4x^2}$ is shown.



Using the result in (a), or otherwise, find the area enclosed between the curve, the x-axis and the line $x = 1$, giving your answer correct to two decimal places. (11)

25 marks

Total for Section A: 200 marks

SECTION B MATHEMATICS OF FINANCE

QUESTION 1

(a) Explain the difference between interest and inflation. (2)

(b) Is compound interest always better than simple interest?
Refer to sketch graphs in motivating your answer. (4)

(c) Explain the difference between a nominal interest rate and an effective interest rate.
Which is more useful to the investor? (4)

10 marks

QUESTION 2

At a certain point in time, a car's value is R84 676,86 and now, after a further 3 years, its value is R53 859,23. Calculate:

(a) the annual rate of depreciation on a reducing balance, correct to the nearest whole percentage. (8)

(b) the current age of the car (to the nearest year), given that its original value was R180 000. (8)

16 marks

QUESTION 3

A lump sum, P, is deposited into an account, which offers interest at 6% per annum compounded quarterly for the first three years and 10% per annum compounded monthly for the next 5 years. Calculate the effective interest rate per annum over the entire 8 year period.

10 marks

QUESTION 4

Joshua takes out a loan against a new house. He requires a bond of R750 000 and he is given a fixed interest rate of 14% per annum over a period of 15 years. He will repay the loan with equal monthly instalments starting one month from when the loan was granted.

(a) Calculate the monthly instalment. (7)

(b) Calculate the outstanding balance after 8 years. (9)

(c) Explain clearly why there is still so much left to pay after 8 years. (2)

(d) What is the total amount of interest paid for the duration of the loan (15 years)? (5)

Suppose Joshua chooses to pay R12 000 per month, which is more than the minimum payment required.

(e) By how many months will the time-span of the loan be reduced? (10)

33 marks

QUESTION 5

Mr Masinga starts a monthly savings plan so that his son, Vusi, can buy a car when he turns 18, in 10 years' time. He deposits a fixed amount each month, starting in one month's time, in an account offering 9,5% per annum compounded monthly. The payments will cease in 6 years' time. In 10 years' time he plans to buy a car which is currently valued at R150 000.

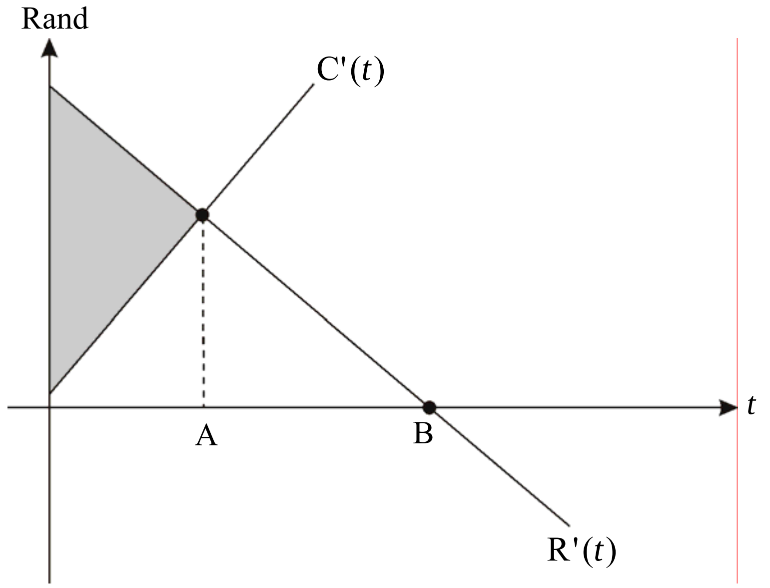
(a) Assuming that inflation will remain constant at 7% per annum over the next 10 years, calculate how much Mr Masinga will need in order to buy the car for Vusi. (6)

(b) Calculate the monthly payment necessary over the next 6 years if Mr Masinga is to reach his target amount at the end of 10 years. (15)

21 marks

QUESTION 6

A graph showing the marginal revenue function, $R'(t)$, and the marginal cost function, $C'(t)$, of a mining company is drawn below. The time, t, is measured in years.



(a) What is the significance of points A and B? (4)

(b) What does the shaded area enclosed between the *y*-axis and the two graphs represent? (3)

(c) Now sketch a possible graph of P(t), the total profit after t years. (3)

10 marks

Total for Section B: 100 marks

SECTION C ALGEBRA

QUESTION 1

(a) Use the method of induction to prove that

$$2\,(1!) + 5\,(2!) + 10\,(3!) + ... + (n^2 + 1)\,n! = n\,(n+1)!$$

for all $n \in N$ (16)

(b) Hence find the sum of the first seven terms. (4)

20 marks

QUESTION 2

$p(x) = x^4 + 3x^3 - 2x^2 - 12x - 8$
$q(x) = x^3 - 7x - 6$

Use the Euclidean algorithm to show that a greatest common divisor (highest common factor) of $p(x)$ and $q(x)$ is $x^2 + 3x + 2$.

10 marks

QUESTION 3

a, b and c are the roots of $x^3 - 3x^2 + mx + 24 = 0$ and $-a$ and $-b$ are the roots of $x^2 + nx - 6 = 0$.

Find the values of m and n.

13 marks

QUESTION 4

(a) Given $f(x) = 2x^3 - 9x^2 + 8x - 2$

(i) Use the rational roots theorem to find all potential zeros of $f(x)$. (6)

(ii) Factorise $f(x)$ fully in $Q\,[x]$. (5)

(b) Use Eisenstein's Criterion to prove whether $p(x) = x^4 - 6x^3 + 12x^2 - 18x - 36$ is irreducible in $Z[x]$. (6)

17 marks

QUESTION 5

Given the rational function $f(x) = \dfrac{ax+b}{cx+d}$ where a, b, c and d are constants, with $c \neq 0$ and $ad \neq bc$.

Find:

(a) the equation(s) of the horizontal asymptote(s) (4)

(b) the equation(s) of the vertical asymptote(s) (2)

6 marks

QUESTION 6

Given that $p(x) = \dfrac{3x^3 - 12x}{x^2 - 1}$

(a) Decompose $p(x)$ into partial fractions. (14)

(b) Find:

(i) any vertical, horizontal or oblique asymptotes to $p(x)$. (4)

(ii) the values of x for which $p(x) > 0$. (8)

(c) Given $p'(x) = \dfrac{3x^4 + 3x^2 + 12}{\left(x^2 - 1\right)^2}$

(i) Determine any stationary points of $p(x)$. (4)

(ii) Hence, or otherwise, determine for what values of x, $p(x)$ is an increasing function. (4)

34 marks

Total for Section C: 100 marks

SECTION D STATISTICS

QUESTION 1

The standard deviation for the number of examination scripts marked by a sub-examiner at the end of the year is 2,87. The average, sampled from 50 sub-examiners, is 35 scripts marked per hour .

5 people (A, B, C, D and E) apply to be markers and during their interview they marked 104; 107; 102; 108 and 106,5 scripts respectively in a three hour session.

The IEB Management decides to hire any applicant whose production is within the 95% confidence range.

Which of the applicants A, B, C, D or E get hired by the IEB?

11 marks

QUESTION 2

Tennis balls dropped on to a concrete floor from a given height, rebound to heights, on the first bounce, which can be modelled by the Normal Distribution with mean 0,8 metres and standard deviation 0,2 metres.

The balls are classified in order of increasing height into 5 categories: Rejected; Super Slow; Slow; Medium; Fast.

(a) Given that 9% of the balls are classified as rejected, calculate the maximum height of rebound of a ball that is classified as rejected. (8)

(b) If the maximum height of rebound of a slow ball is 0,9 metres, what percentage of balls are either 'Slow' or 'Super Slow'? (4)

(c) If the percentages of balls classified as Medium and Fast are equal, calculate the minimum height of rebound of a ball classified as Fast. (8)

20 marks

QUESTION 3

It is widely believed that 78% of all visitors to South Africa during the World Cup Soccer in 2010 will also visit a game farm during their stay here. What size sample will be necessary to get this estimate correct to within 5% with 90% confidence?

12 marks

QUESTION 4

Nine men are lined up in an identity parade which includes three burglary suspects.
A witness must try to identify the three burglary suspects.

(a) How many different arrangements are possible with the three suspects standing next to each other? (4)

(b) How many different arrangements are possible with no two of the suspects standing next to each other? (7)

11 marks

QUESTION 5

There are 7 white discs, 10 black discs and 3 green discs in a bag.
5 discs are selected at random.

Find the probability that the sample of 5 will

(a) include at least two that are white, if the selection is done with replacement. (8)

(b) include the same number of green and white discs, if the selection is done without replacement. (10)

18 marks

QUESTION 6

A survey of 500 tourists to South Africa revealed the following data:

329 visited Cape Town
186 visited the Kruger Park
295 visited Sun City
83 visited Cape Town and the Kruger Park
217 visited Cape Town and Sun City
63 visited the Kruger Park and Sun City

If every tourist visited at least one of these destinations, determine how many tourists visited all three destinations.

12 marks

QUESTION 7

The manager of a league soccer team contacted the local meteorological office regarding the likely weather for the opening day of the season and was given the table of probabilities below:

Wind speed / Atmosphere	Less than 10 km/h	Between 10 km/h and 30 km/h	More than 30 km/h
Sunny	0,30	0,21	0,09
Partly Cloudy	0,14	x	0,06
Raining	0,06	0,05	0,02

(a) Find the probability that:

(i) the opening day will be sunny. (2)

(ii) the wind speed will be less than 30 km/h. (3)

(iii) it will rain or the wind speed will be less than 10km/h. (3)

(b) Show that the probability, x, of the atmosphere being partly cloudy and wind speed between 10 km/h and 30 km/h is 0,07. (3)

(c) Show that the data suggests that the event of sunny weather is independent of a wind speed of less than 10km/h. (5)

16 marks

Total for Section D: 100 marks

Grand Total: 400 marks