



SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2007

ADDITIONAL MATHEMATICS HIGHER GRADE

MEMORANDUM

Time: 3 hours

400 marks

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SECTION A CALCULUS

QUESTION 1

(a) $f(x) = \frac{(3x-1)(x-1)}{(1-x)(1+x)} = \frac{-(3x-1)}{x+1}$

(i) $\lim_{x \rightarrow 0} f(x) = 1$ ✓ sub ✓ answer or answer only ✓ ✓ (2)

(ii) $\lim_{x \rightarrow 1} f(x) = \frac{-2}{2} = -1$ ✓ factorise ✓ cancel ✓✓ if answer = +1 → 3 marks (4)

(iii) $\lim_{x \rightarrow \infty} f(x) = \frac{3 - \frac{4}{x} + \frac{1}{x^2}}{\frac{1}{x^2} - 1} = -3$ technique ÷ by x^2 ✓✓
 answer ✓✓ if answer = +3 → 3 marks (4)

✓✓ idea $\lim_{x \rightarrow 0} \frac{(x+h)^2 + 1}{h(\sqrt{(x+h)^2 + 1} + \sqrt{x^2 + 1})}$ ✓✓ idea of rationalising
 ✓✓ first step

(b) $\frac{1}{2}(x^2+1)^{\frac{1}{2}} \cdot 2x$ or $\lim_{h \rightarrow 0} \frac{2xh+h^2}{h(\sqrt{(x+h)^2 + 1} + \sqrt{x^2 + 1})}$ ✓✓ simplifying
 $= \frac{x}{\sqrt{x^2 + 1}}$ ✓

(7)

17 marks

QUESTION 2

$-6 + 15 = 3ab + 3$
 $6 = 3ab$
 $2 = ab$

and

$9 = 4a + b$
 $9 - 4a = b$
 $2 = a(9 - 4a)$ ✓ m
 $4a^2 - 9a + 2 = 0$ ✓ c/a
 $(a - 2)(4a - 1) = 0$ ✓ c/a
 $a = 2; a = \frac{1}{4}$ ✓✓ c/a
 $b = 1; b = 8$ ✓✓ c/a

✓✓✓✓ method showing principle of LH and RH limit

16 marks

QUESTION 3

- (a) $\theta = \arcsin \frac{6}{8}$ ✓✓
 $\theta = 0,848062079$ ✓✓
 $\cos(\pi/2 - \theta) = \frac{6}{8}$
 $48,59^\circ / 0,8 / 0,85 / 0,848$ 3 marks
- (b) $Area\ ABCD = 60m^2$ ✓ rectangle
 $Area\ sector = \frac{1}{2}r^2\theta$ ✓ formula (4)
 $area = \frac{1}{2}(8)^2(\arcsin \frac{6}{8})$ ✓✓ sub
 $= 27,14$ ✓ c/a answer ✓ method
 $Area\ \Delta APD = \frac{1}{2}(8)(6)\sin \frac{\pi}{2} \arcsin \frac{6}{8}$ ✓^m ✓^{ca} ✓^a OR $Area\ of\ \Delta APD = \frac{1}{2}(6)(\sqrt{28})$
 $= 15,8745$ ✓ method
 $\square^{final}\ area = 16,9875m^2$ ✓ ca ✓ acc (10)
 If eating area is found → 8 marks ✓ answer

14 marks

QUESTION 4

- (a) $(3a - 2)^2 = 100$ sub one into other ✓
 $3a - 2 = \pm 10$ let $x = a$ (no mark)
 $a = 4$ or $a = -2\frac{2}{3}$ ✓ equate to 100 ✓ (4)
- (b) $(3b - 2)^2 = 3b^2 - 2$ ✓^{sub}
 $9b^2 - 12b + 4 = 3b^2 - 2$ ✓ simplify
 $6b^2 - 12b + 6 = 0$ ✓ simplify ca
 $6(b - 1)^2 = 0$ ✓ c/a
 $b = 1$ ✓ c/a (6)

10 marks

QUESTION 5

(a) $f'(x) = 3ax^2 + 2bx + c$ ✓ a ✓ a
 $f''(x) = 6ax + 2b$ ✓✓ c/a (4)

(b) $f'(x)$ is a quadratic/parabola which may or may not have real roots therefore may not have stationary values, but $f''(x)$ is a linear function which has to have a solution for x. ✓✓ ✓✓ (4)

(c) $f'(x) = 3ax^2 + 2bx + c = 0$

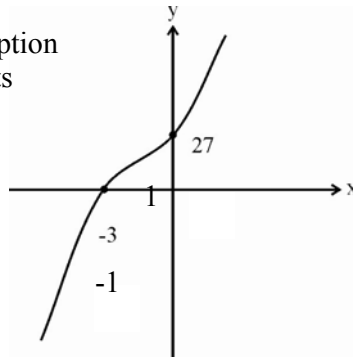
$$x = \frac{-b \pm \sqrt{b^2 - 3ac}}{3a}$$

choose $b = 1, a = 1, c = 1$ $\Delta < 0$ (6)

$y = x^3 + x^2 + x + 1$

$x^2(x+1) + (x+1) = 0$
 $(x+1)(x^2+1) = 0$
 $x = -1$

- ✓✓ sketch graph of this description
- ✓✓ quadratic doesn't have roots
- ✓✓ cubic having integrated



14 marks

QUESTION 6

(a) $f'(x) = \frac{1}{2}(\sqrt{6x+12} - \sqrt{x})^{-\frac{1}{2}} \times \left(\frac{1}{2}(6x+12)^{-\frac{1}{2}} \times 6 - \frac{1}{2}x^{-\frac{1}{2}} \right)$

sub
 $f'(4) = \frac{1}{2}(6-2)^{-\frac{1}{2}} \times \left(\frac{1}{2} - \frac{1}{4} \right)$

$\therefore f'(4) = \frac{1}{16}$ ✓✓ c/a (10)

(b) $f'(x) = \frac{\sin x \cdot x - (10 - \cos x)}{x^2}$ now
 $= \frac{x \sin x - 10 + \cos x}{x^2}$

$\frac{x^2 \sin x - 10x + x \cos x}{x^2} + \frac{10 - \cos x}{x}$ ✓ multiply process ✓
 $= \sin x - \frac{10}{x} + \frac{\cos x}{x} + \frac{10}{x} - \frac{\cos x}{x}$ ✓✓
 $= \sin x$
 $\frac{7}{10}$ if don't multiply through (10)

20 marks

QUESTION 7

$$\begin{aligned}
 \text{(a)} \quad \frac{d}{dx}(\arctan x) &= \frac{d}{dx} \left(x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \frac{x^9}{9} - \dots \right) \quad \checkmark\checkmark \text{ understanding method} \\
 &= 1 - \frac{3x^2}{3} + \frac{5x^4}{5} - \frac{7x^6}{7} + \frac{9x^8}{9} - \dots \quad \checkmark\checkmark \\
 &= 1 - x^2 + x^4 - x^6 + x^8 - \dots \quad \checkmark
 \end{aligned}$$

But this is an infinite geometric series with $r = -x^2$ ✓

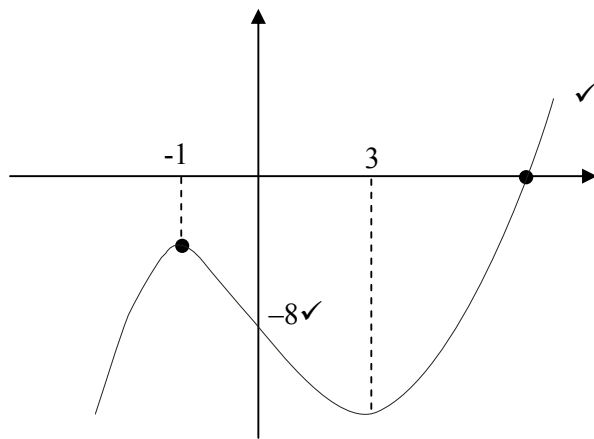
Formula ✓✓

$$\therefore S_\infty = \frac{1}{1-r} \quad a=1 \quad \checkmark$$

$$= \frac{1}{1-(-x^2)} \quad \checkmark \text{ tidy up}$$

$$= \frac{1}{1+x^2} \quad (10)$$

(b) (i) $f'(x) = 3x^2 - 6x - 9$ ✓
 $f'(x) = 3(x+1)(x-3) = 0$ ✓ solving
 $x = -1; x = 3$ ✓✓ (6)



(ii) increasing after $x = 3$ ✓✓ notion

$$a_0 = 3,2$$

$$a_1 = 3,2 - \frac{(3,2)^3 - 3(3,2)^2 - 9(3,2) - 8}{3(3,2)^2 - 6(3,2) - 9} \quad \begin{array}{l} \checkmark \text{ formula} \\ \checkmark \checkmark \text{ derivative} \end{array}$$

$$= 10,6412$$

$$a_2 = 7,727781092$$

$$a_3 = 5,944211992$$

$$a_4 = 4,989924854$$

$$a_5 = 4,636484093$$

$$a_6 = 4,584044293$$

$$a_7 = 4,582919183$$

$$a_8 = 4,58291867$$

$$\therefore \text{root} = 4,5829$$

$\xrightarrow{5,0811}$ ✓✓✓ answer
correct number of decimal places

(8)

24 marks**QUESTION 8**

$$f(x) = 3x^{-1} \quad \checkmark \text{ correct form}$$

$$f'(x) = -1(3)x^{-2} \quad \checkmark \checkmark$$

$$f''(x) = 1(3)(2)x^{-3} \quad \checkmark \checkmark$$

$$f'''(x) = -1(3)(2)(3)x^{-4}$$

$$f^n(x) = (-1)^n \cdot 3 \cdot n! \cdot x^{-(n+1)} \quad \checkmark$$

10 marks**QUESTION 9**

$$(a) \quad \hat{A} + \hat{B} + \hat{C} = 180 \quad \checkmark \checkmark \quad (\text{Ls of a } \triangle)$$

$$A + \frac{2B}{\pi} = 180 \quad \checkmark \quad (\text{sub c})$$

$$A = 180 - \frac{2B}{\pi} \quad \checkmark \quad \text{rearrange}$$

$$\cos A = \cos(180 - 2B) \quad \checkmark$$

$$\cos A = -\cos 2B \quad \checkmark$$

(6)

(b) $\cos A + \cos B = -\cos 2B + \cos B$ ✓ sub for cos A → - cos 2B
 $\sin 2B \cdot 2 - \sin B = 0$ ✓
 $2\sin B \cos B \cdot 2 - \sin B = 0$ double angle
 $\sin B(4\cos B - 1) = 0$ factorise
 $\sin B = 0$ or $\cos B = \frac{1}{4}$

$B = 0E; \pi$ or $B = 1,318116$ ✓✓

□ $\cos A + \cos B$

$= -\cos 2(1,318116) + \cos(1,318116)$ ✓^m✓^m sub into correct exp.

$= \frac{9}{8}$ ✓ ca

(14)

20 marks

QUESTION 10

$\left[4x - x^3 \right]_{-1}^{2k} = 3$

✓ sub 2K ✓ subtract

✓ sub -1 ✓ =3

$8k - 8k^3 + 4 - 1 = 3$

$8k - 8k^3 = 0$

✓ simplifying

$\therefore k = 0, k = 1, k = -1$
 ✓ca ✓ca ✓ca

10 marks

QUESTION 11

(a) $\frac{1}{2} \int (2x+4)(x^2+4x+5)^{\frac{1}{2}} dx$
 $= \frac{1}{2} \frac{(x^2+4x+5)^{\frac{3}{2}}}{\frac{3}{2}} + C$ ✓
 $= \frac{(x^2+4x+5)^{\frac{3}{2}}}{3} + C$

(b) $\int x \cos 2x dx$
 $= x \cdot \frac{\sin 2x}{2} - \int \frac{\sin 2x}{2} 1 dx$
 $= \frac{x \sin 2x}{2} + \frac{\cos 2x}{4} + C$

let $u = x^2 + 4x + 5$ ✓ sub
 $\frac{du}{dx} = (2x + 4)$ ✓ derivative

$du = 2(x+2) dx$

$\int (x+2) u^{\frac{1}{2}} \cdot \frac{du}{2(x+2)}$ ✓ (9)

$= \frac{1}{2} \int u^{\frac{1}{2}} du$ ✓

$= \frac{1}{2} \cdot \frac{u^{\frac{3}{2}}}{\frac{3}{2}} \times 2 + C$ (11)

20 marks

$\int f(x) g'(x) = f(x)g(x) - \int f'(x) \cdot g(x) dx$

$= \frac{u^{\frac{3}{2}}}{3} + C$

$f(x) = x$ ✓ $f'(x) = 1$ ✓
 $g'(x) = \cos 2x$ ✓ $g(x) = \frac{\sin 2x}{2}$ ✓

$= \frac{(x^2+4x+5)^{\frac{3}{2}}}{3} + C$ ✓

QUESTION 12

(a) $\frac{dx}{d\theta} = 2 \cos \theta \checkmark$. $dx = 2 \cos \theta d\theta$

$$\int \sqrt{4 - 4 \sin^2 \theta} \cdot 2 \cos \theta d\theta \quad \sqrt{(4 - (2 \sin \theta)^2)} \checkmark$$

$$= \int 2 \sqrt{\cos^2 \theta} \cdot 2 \cos \theta d\theta \quad = \sqrt{4 - 4 \sin^2 \theta} \checkmark$$

$$= \int 4 \cos^2 \theta d\theta \quad | \text{ for square rooting } \cos^2 \theta \text{ and other for multiplying it by } \cos \theta.$$

$$= \int (2 \cos 2\theta + 2) d\theta$$

$$= \sin 2\theta + 2\theta + C \quad (14)$$

(b) $y = \sqrt{16 - 4x^2} = 2\sqrt{4 - x^2}$ Area = $\int_1^2 \sqrt{16 - 4x^2} dx$

$$\text{Area} = 2 \int_1^2 \sqrt{4 - x^2} dx$$

$$= 2 [\sin 2\theta + 2\theta]_{\theta = \frac{\pi}{6}}^{\theta = \frac{\pi}{2}}$$

replacement
method idea of changing limits

$$= 2,46 \text{ units}^2 \checkmark$$

cla.

$$x = 2 \sin \theta$$

$$2 \sin \theta = 1$$

$$\sin \theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{6}$$

$$2 \sin \theta = 2$$

$$\sin \theta = 1$$

$$\theta = \frac{\pi}{2} \quad (11)$$

25 marks

Total: 200 marks

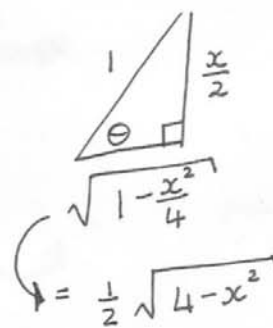
$$y = \sin 2\theta + 2\theta + C$$

$$\frac{dy}{d\theta} = 2 \cos 2\theta + 2$$

$$= 2 (2 \cos^2 \theta - 1) + 2$$

$$= 4 \cos^2 \theta$$

$$= \sqrt{4 - x^2}$$



$$\sin \theta = \frac{x}{2}$$

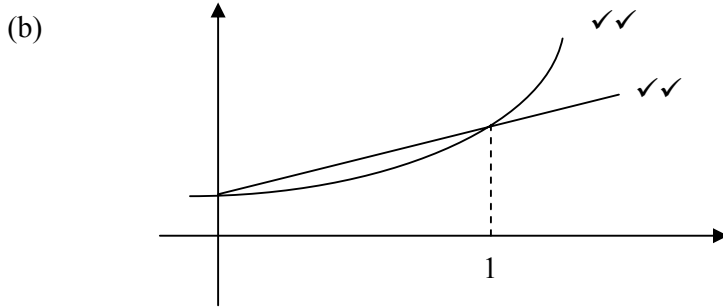
$$\cos^2 \theta = \left(\frac{1}{2} \sqrt{4 - x^2} \right)^2$$

$$= \left(\frac{\sqrt{4 - x^2}}{2} \right)^2 = \frac{4 - x^2}{4}$$

SECTION B MATHEMATICS OF FINANCE

QUESTION 1

- (a) Interest is the return on money invested. a ✓
 Inflation is the gradual rise in prices of a cross-section of commodities. a ✓ (2)



For the first year simple interest is slightly better than compound interest. Thereafter, compound interest is much better. (4)

- (c) Nominal interest is stated ✓ with a period of compounding. Effective ✓ interest is the actual % increase in the investment, which is more useful to know. (4)
✓✓ effective

10 marks

QUESTION 2

(a) $53859,23 ✓ = ✓ 84676,86(1 - i)^3 ✓$ ✓✓
 annual rate = 14% ✓✓ ✓ (8)

(b) $84676,86 ✓ = ✓ 180000(1 - 0,14)^n ✓$ ✓✓
 $n = 5$, therefore the car is now 8 years old. (8)
✓✓ ✓

16 marks

QUESTION 3

$P(1 + i)^8 ✓✓ = P \left(1 + \frac{0,06}{4}\right)^{12} \left(1 + \frac{0,1}{12}\right)^{60}$ ✓ ✓ ✓ ✓ ✓ mult
 $(1 + i)^8 = \left(1 + \frac{0,06}{4}\right)^{12} \left(1 + \frac{0,1}{12}\right)^{60} ✓$
 $i = 8,83%$ ✓✓

10 marks

QUESTION 4

$$\begin{aligned} \text{(a) } \text{reff} &= \left(1 + \frac{r}{n}\right)^{12-1} \\ 0,14 &= \left(1 + \frac{r}{12}\right)^{12-1} \quad \checkmark\checkmark \\ r &= 0,1317 \end{aligned}$$

$$750\,000 = x \left[\frac{1 - \left(1 + \frac{0,1317}{12}\right)^{-180}}{\frac{0,1317}{12}} \right] \quad \checkmark^m$$

$$x = 9573,36 \quad \checkmark$$

$$\begin{aligned} \text{(b) } O.B. &= 9573,36 \left[\frac{1 - \left(1 + \frac{0,1317}{12}\right)^{-84}}{\frac{0,1317}{12}} \right] \quad \checkmark^m \checkmark^m \checkmark^m \\ &= 523577,80 \quad \checkmark\checkmark\checkmark \end{aligned} \tag{7}$$

(c) During the first half of the term of the loan most of the payments go towards paying the interest which is large because the O.B. is at its largest. The O.B. reduces much more quickly during the last half of the period of the loan. (2)

$$\begin{aligned} \text{(d) } 180 \times 9573,36 &- 750\,000 \\ &= 973204,80 \quad \checkmark \end{aligned} \tag{5}$$

$$\begin{aligned} \text{(e) } 750\,000 &= 12\,000 \left[\frac{1 - \left(1 + \frac{0,1317}{12}\right)^{-n}}{\frac{0,1317}{12}} \right] \quad \checkmark\checkmark\checkmark \\ n &= 107 \text{ months. Therefore loan is reduced by } 73 \text{ months.} \end{aligned} \tag{10}$$

33 marks

QUESTION 5

$$\begin{aligned} \text{(a) } F &= 150\,000(1 + 0,07)^{10} \quad \checkmark^m \\ &= R295\,072,70 \quad \checkmark\checkmark \end{aligned} \tag{6}$$

$$\begin{aligned} \text{(b) } x \left[\frac{\left(1 + \frac{0,095}{12}\right)^{72} - 1}{\frac{0,095}{12}} \right] &= 295\,072,70 \quad \checkmark^m \checkmark^m \checkmark^m \quad \checkmark^m \checkmark^m \\ x &= R2\,093,32 \quad \checkmark\checkmark\checkmark \end{aligned} \tag{15}$$

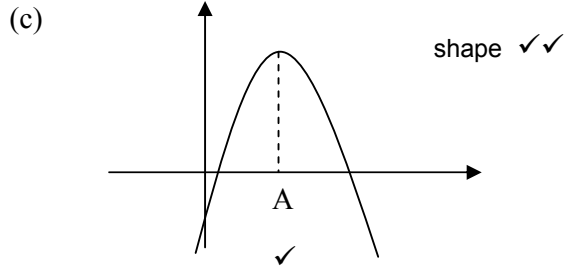
21 marks

QUESTION 6

(a) A is the no. of years which gives maximum profit. ✓✓

B is the number of years which gives maximum revenue. ✓✓ (4)

(b) Total profit ✓✓, without including fixed ✓ costs. (3)



(3)

10 marks

Total: 100 marks

QUESTION 2

$$p(x) = x^4 + 3x^3 - 2x^2 - 12x - 8$$

long division
✓✓

$$q(x) = x^3 - 7x - 6 \quad \checkmark\checkmark$$

$$\begin{array}{r} x^3 - 7x - 6 \overline{) x^4 + 3x^3 - 2x^2 - 12x - 8} \\ \underline{x^4 - 7x^2 - 6x} \\ 3x^3 + 5x^2 - 6x - 8 \\ \underline{3x^3 - 21x - 18} \\ 5x^2 + 15x + 10 \quad \checkmark \end{array}$$

$$\begin{array}{r} \overline{) x^3 + 0x^2 - 7x - 6} \quad \checkmark \\ \underline{x^3 + 3x^2 + 2x} \\ -3x^2 - 9x - 6 \\ \underline{-3x^2 - 9x - 6} \\ 0 \end{array} \quad \checkmark$$

$$5x^2 + 15x + 10$$

Last non-zero remainder is a GCD : $5(x^2 + 3x + 2)$

$$\therefore x^2 + 3x + 2 \quad \checkmark\checkmark\checkmark$$

10 marks

QUESTION 3

$$x^3 - 3x^2 + mx + 24 = 0 \text{ and } x^2 + nx - 6 = 0.$$

$$SOR = a + b + c = 3 \quad \checkmark$$

$$\text{and } SOR = -a + -b = -n \quad \checkmark$$

$$POR = abc = -24 \quad \checkmark$$

$$(a + b) = n \quad \checkmark$$

$$ab + ac + bc = m \quad \checkmark$$

$$POR = (-a)(-b) = -6 \quad \checkmark$$

$$ab = -6 \dots\dots * \quad \checkmark$$

$$\text{now } (-6)c = -24(\dots) \quad \checkmark$$

$$c = 4 \quad \checkmark$$

$$\therefore n + 4 = 3 \quad \checkmark$$

$$n = -1 \quad \checkmark$$

$$\therefore -6 + c(a + b) = m \quad \checkmark$$

$$-6 + 4(-1) = m$$

$$-10 = m \quad \checkmark$$

13 marks

QUESTION 4

(a) (i) $f(x) = 2x^3 - 9x^2 + 8x - 2$
 $r = \pm 1; \pm 2$
 $p = \pm 1; \pm 2$
 $\therefore \frac{r}{p} = \pm 1; \pm \frac{1}{2}; \pm 2 \quad \checkmark \checkmark$

(6)

By factor theorem $f\left(\frac{1}{2}\right) = 0$

(ii) $(2x - 1)$ is a factor
 $\therefore (2x - 1)(x^2 - 4x + 2) \quad \checkmark \checkmark$

(5)

(b) $p(x) = x^4 - 6x^3 + 12x^2 - 18x - 36$

Try $p = 3$ or 2

- p does not divide into coefficient of leading term
 - p does divide into all other coefficients
 - p^2 does divide into constant
 - \therefore can't conclude that $p(x)$ is irreducible in $Z[x]$
- No conclusion possible.

(6)

17 marks

QUESTION 5

(a) $\lim_{x \rightarrow \infty} \frac{ax + b}{cx + d}$
 $= \lim_{x \rightarrow \infty} \frac{x(a + \frac{b}{x})}{x(c + \frac{d}{x})}$ same for $x \rightarrow -\infty$
 $= \frac{a}{c}$

(4)

(b) $cx + d = 0$
 $x = \frac{-d}{c}$

(2)

6 marks

QUESTION 6

(a) $p(x) = \frac{3x^3 - 12x}{x^2 - 1}$

$$\begin{array}{r} 3x \\ x^2 - 1 \overline{) 3x^3 - 12x} \\ \underline{3x^3 - 3x} \\ -9x \end{array}$$

$$\frac{3x^3 - 12x}{x^2 - 1} = 3x - \frac{9x}{x^2 - 1}$$

$$\frac{9x}{x^2 - 1} = \frac{A}{x-1} + \frac{B}{x+1}$$

$$9x = A(x+1) + B(x-1)$$

let $x = 1$

$$9 = 2A$$

Now $A = \frac{9}{2}$

let $x = -1$

$$-9 = -2B$$

$$B = \frac{9}{2}$$

Now $\frac{3x^3 - 12x}{x^2 - 1} = 3x - \frac{9}{2(x-1)} - \frac{9}{2(x+1)}$ (14)

- (b) (i) vertical asymptotes at $x = 1$ and $x = -1$
 horizontal: none
 oblique asymptote at $y = 3x$ (4)

(ii) $\frac{3x(x-2)(x+2)}{(x-1)(x+1)} > 0$ (8)
 $-2 < x < -1$ or $0 < x < 1$ or $2 < x < \infty$

$$(c) (i) \quad p'(x) = \frac{3x^4 + 3x^2 + 12}{(x^2 - 1)^2}$$

$$3x^4 + 3x^2 + 12 = 0$$

$$x^4 + x^2 + 4 = 0$$

$$\Delta = 1 - 4(1)(4)$$

$$\Delta < 0$$

\therefore no real roots

\therefore no stationary values

(4)

$$(ii) \quad p'(x) = \frac{3x^4 + 3x^2 + 12}{(x^2 - 1)^2} \quad \text{but} \quad p'(x) = \frac{3(x^4 + x^2 + 4)}{(x^2 - 1)^2} \quad \text{which is always positive,}$$

therefore $x \in R ; x \neq \pm 1$

(4)

↑
note

34 marks

Total: 100 marks

SECTION D STATISTICS

QUESTION 1

$\sigma = 2,87$

$\bar{x} = 35$

$n = 50$

$\bar{x} \pm 1,96 \frac{\sigma}{\sqrt{n}}$
 $= 35 \pm 1,96 \cdot \frac{2,87}{\sqrt{50}}$ m✓

$= 35 \pm 0,79552341$

$= 35,796$ ✓ or $34,204$ ✓

$A = 34,8$; $B = 35,6$

$C = 34$; $D = 36$ ✓

$E = 35,5$

$\therefore A, B, E$ get hired

11 marks

QUESTION 2

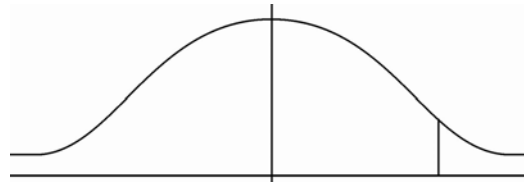
(a) $0,5 - 0,09 = 0,41$ ✓ ✓

$H(z) = 0,41$

$z = -1,34$ ✓ ✓

$-1,34 = \frac{x - 0,8}{0,2}$ ✓

$0,532 = x$ ✓ ✓ (1,068 in ve Z)



Therefore maximum height reached by ball classified as rejected is 0,532m. (8)

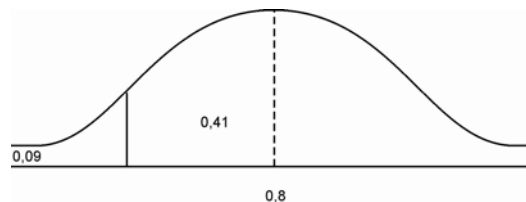
(b) $x = 0,9$

$z = \frac{0,9 - 0,8}{0,2}$

$z = 0,5$

$H(z) = 0,1915$

now $0,41 + 0,1915$
 $= 0,6015$



$\therefore 60,15\%$ of balls are slow or super slow (4)

$$\begin{aligned}
 \text{(c)} \quad & 0,5 - 0,1915 \\
 & = 0,3085 \\
 & \therefore \frac{0,3085}{2} = 0,15425 \\
 & \text{now } 0,15425 + 0,1915 \\
 & = 0,34575 \\
 & z = 1,02 \quad \checkmark \\
 & 1,02 = \frac{x - 0,8}{0,2} \quad \checkmark \\
 & x = 1,004m \quad \checkmark
 \end{aligned}$$

(8)

20 marks

QUESTION 3

$$\begin{aligned}
 & p \pm 1,645 \sqrt{\frac{p(1-p)}{n}} \quad \checkmark \checkmark \\
 & 1,645 \sqrt{\frac{0,78(0,22)}{n}} < 0,05 \quad \checkmark \\
 & \frac{0,78(0,22)}{n} < \left(\frac{0,05}{1,645}\right)^2 \\
 & n > \left(\frac{1,645}{0,05}\right)^2 \times 0,78 \times 0,22 \\
 & n > 185,74 \quad \checkmark \text{ rounding} \\
 & n = 186 \quad \checkmark \checkmark \checkmark
 \end{aligned}$$

12 marks

QUESTION 4

(a) 6 ordinary men; 3 suspects

$$7! \times 3! = 30240$$

(4)

(b) ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓

$$6! \times {}^7C_3 \times 3! = 151200$$

(7)

11 marks

QUESTION 5

7 white; 3 green; 10 black

$$(a) \quad 1 - [{}^5C_0 \left(\frac{7}{20}\right)^0 \left(\frac{13}{20}\right)^5 + {}^5C_1 \left(\frac{7}{20}\right)^1 \left(\frac{13}{20}\right)^4]$$

$$= 0,571585$$

(8)

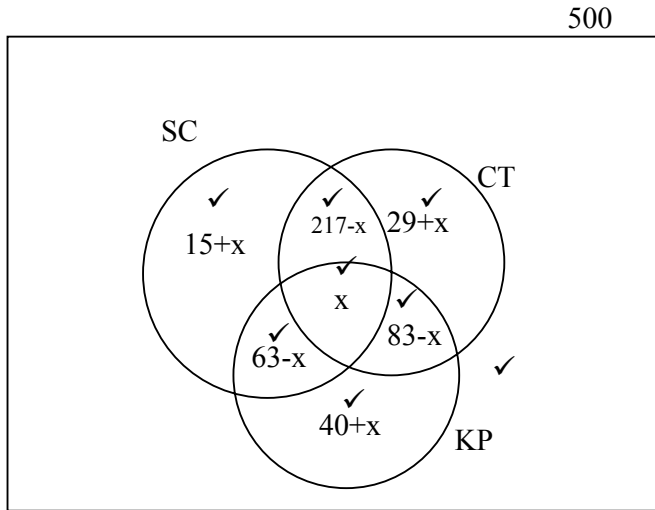
$$(b) \quad \frac{\binom{7}{1} \binom{3}{1} \binom{10}{3}}{\binom{20}{5}} + \frac{\binom{7}{2} \binom{3}{2} \binom{10}{1}}{\binom{20}{5}} + \frac{\binom{10}{5} \binom{10}{0}}{\binom{20}{5}}$$

$$= 0,2194272446$$

(10)

18 marks

QUESTION 6



$$15 + x + 217 - x + 29 + x + 63 - x + x + 83 - x + 40 + x = 500 \quad \checkmark\checkmark$$

$$447 + x = 500$$

$$x = 53 \quad \checkmark\checkmark$$

12 marks

QUESTION 7

- (a) (i) 0,6 $\checkmark\checkmark$ (2)
- (ii) $1 - (0,09 + 0,06 + 0,02) = 0,83 \quad \checkmark$ (3)
- (iii) 0,57 $\checkmark\checkmark\checkmark$ (3)
- $\checkmark\checkmark\checkmark$
- (b) $1 - (0,93) = 0,07$ (3)

$P(S) = 0,6$

$P(\text{wind} < 10\text{km}) = 0,5$

(c) $P(S) \times P(\text{wind} < 10\text{km}) = 0,6 \times 0,5 \quad \checkmark\checkmark$

$= 0,3$

but $P(S \cap W) = 0,3 \quad \checkmark$

So the data suggests that they are independent events. (5)

16 marks

Total: 100 marks

Grand Total: 400 marks